Problema 9-3
Theory of Equations
Uspensky

Eduardo Virueña Silva

Noviembre, 2025

Problema 1. *Muestre que, para cualesquiera dos números $x, y \in \mathbb{C}$:*

$$|x+y|^2 + |x-y|^2 = 2|x|^2 + 2|y|^2$$

Demostración.

$$\begin{aligned} |x+y|^2 &= (x+y)\overline{(x+y)} = (x+y)(\bar{x}+\bar{y}) \\ &= x\bar{x} + x\bar{y} + \bar{x} + y\bar{y} \\ &= |x|^2 + x\bar{y} + \bar{x}y + |y|^2 \\ |x-y|^2 &= (x-y)\overline{(x-y)} = (x-y)(\bar{x}-\bar{y}) \\ &= x\bar{x} - x\bar{y} - \bar{x}y + y\bar{y} \\ &= |x|^2 - x\bar{y} - \bar{x}y + |y|^2 \end{aligned}$$

sumando

$$|x+y|^2 + |x-y|^2 = 2|x|^2 + 2|y|^2$$

□